HEAT TRANSFER IN A UNIFORM MIXTURE OF TWO DISPERSED MATERIALS

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Heat transfer in a dispersed system consisting of a uniform mixture of a coarse- and a fine-grained material with boundary conditions of the fourth kind is considered. The solution is compared with experimental data. Expressions estimating the time of onset of thermal quasi-equilibrium are obtained.

The method of solving problems of this type is usually based on a particular ordered structure for the mixture and selecting an elementary cell of the system. By the latter we mean a minimal volume such that if it is duplicated in a particular manner an infinite number of times it reproduces the structure of the mixture. For a system consisting of a uniform mixture of particles corresponding to the condition $d_1 \gg d_2$, this is a cube at the center of which is a particle of dimension d_1. In the limit, the problem can be reduced to the discussion of heat transfer between a spherical source and a semi-infinite mass of dispersed material. A similar problem was investigated in [1, 2] for boundary conditions of the third kind. As a result, the relative coefficients of heat transfer between a sphere and a dispersed medium were determined.

The above results, processed in accordance with the computational recommendations of [2], are shown in Fig. 1. It follows from this that the results are in conflict with each other, a difference in the relation between Nu and d_1 being observed.

Heat transfer in a uniform mixture of two dispersed materials having different temperatures at the initial moment of time was discussed in [8-10]. Thus, Tsukhanova and Salamandra recommend the following equations for the calculation of the temperature of the mixture components:

$$\theta_1 = (t_1^0 - t_2^0)\frac{m}{m+1}[m + \exp(-k\tau)], \tag{1}$$

$$\theta_2 = (t_1^0 - t_2^0)\frac{m}{m+1}[1 - \exp(-k\tau)], \tag{2}$$

$$k = \frac{6\alpha}{c_1\rho_1 d_1}(1+m); \quad m = \frac{\eta_1 c_1}{\eta_2 c_2}; \quad \theta_1 = \frac{t_1 - t_2^0}{t_1^0 - t_2^0}; \quad \theta_2 = \frac{t_2 - t_2^0}{t_1^0 - t_2^0}.$$

Since the heat-transfer coefficient α is unknown in these equations, the authors propose to determine it in advance for each case in experiments on the cooling of a sphere in a dispersed system.

The following equation was obtained in [9] on the basis of experiments on heat transfer in a uniform mixture of two dispersed materials using the data of [8]:

$$\mathrm{Bi} = -0.783 + 0.726\frac{0.5(T_1^0 + T_2^0) + T_k}{2T_0}. \tag{3}$$

The heat-transfer coefficient, computed from Eq. (3), differs from experimental data of [1] in many cases by more than 50 times. This indicates the need to seek other solutions of the problem. We seek the solution for the cases $d_1 \gg d_2$ in the following conditions.

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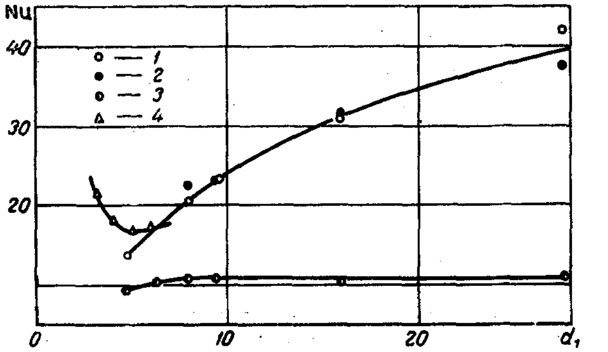

Fig. 1

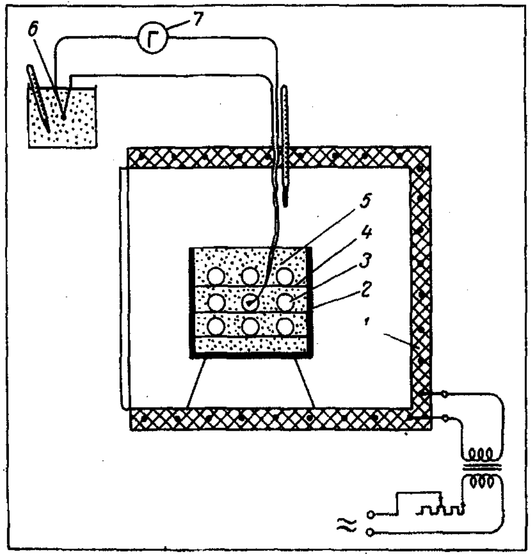

Fig. 2

Fig. 1. The Nusselt number Nu as a function of d_1 (d_1, mm): 1) cooling of a metal sphere in a filling of metal spheres (d_2 = 4.76 mm) in the temperature range 660-250°C [1]; 2) as above in the temperature range 250-100°C [1]; 3) cooling of a metal sphere in a filling of half-coke in the temperature interval 600-250°C [1] (d_2 = 3-5 mm); 4) cooling of a metal sphere in a filling of metal spheres in the temperature interval 350-100°C, $d_1 = d_2$ [2].

Fig. 2. The experimental apparatus: 1) drying cabinet; 2) wooden box; 3) spheres; 4) porcelain grid; 5) sand; 6) cold thermocouple junction; 7) galvanometer.

1. The more dispersed material is assumed to be quasi-homogeneous, subject to the Fourier differential equation in terms of the effective characteristics of the medium.
2. The system of thermal conductivity differential equations is solved with boundary conditions of the fourth kind.

It is assumed that the process satisfies the condition $Fo > Fo_{cr}$, i.e., the time during which the two materials are in contact has no effect on the applicability of condition (1) [3, 4]. That this is permissible is confirmed, in particular, by probe methods of determining the thermophysical characteristics of the dispersed materials [5] and also by the comparison of the analytic solution [6] with experimental results [7].

To simplify the solution we replace the elementary cell – a cube – by an equivalent sphere with a particle d_1 at its center. The effect of the surrounding medium on the boundaries of an elementary volume is determined by the adiabatic condition. In this case the change in the area of the elementary surface as a result of replacing the cube by an equivalent sphere does not play a significant role. For an adiabatic composite sphere, the particles of which have different temperatures at the initial moment of time, the problem can be formulated mathematically as follows:

$$\frac{\partial\theta_1}{\partial\tau} = a_1\left(\frac{\partial^2\theta_1}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_1}{\partial r}\right)_{r<R_1},$$

$$\frac{\partial\theta_2}{\partial\tau} = a_2\left(\frac{\partial^2\theta_2}{\partial r^2} + \frac{2}{r}\frac{\partial\theta_2}{\partial r}\right)_{,\ R_1},$$

boundary conditions

$$\lambda_1\frac{\partial\theta_1}{\partial r} = \lambda_2\frac{\partial\theta_2}{\partial r} \quad \text{for } r = R_1,$$

$$\theta_1 = \theta_2 \quad \text{for } r = R_1 \text{ and } \tau > 0,$$

$$\frac{\partial\theta_1}{\partial r} = 0 \quad \text{for } r = 0,$$

$$\frac{\partial\theta_2}{\partial r} = 0 \quad \text{for } r \geqslant R_2,$$

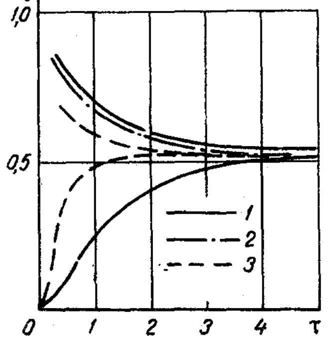

Fig. 3

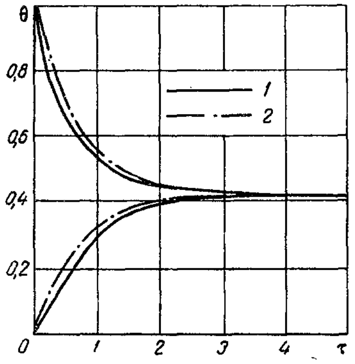

Fig. 4

Fig. 3. Comparison of the calculated and experimental results (τ, min): 1) calculation by numerical method; 2) experimental results; 3) calculation by the method of [1].

Fig. 4. Comparison of the solution by the numerical method with that given by (9), (10) (τ, min): 1) numerical method; 2) from Eqs. (9), (10).

initial conditions

$$\tau = 0, \quad \theta_1 = 1, \quad \theta_2 = 0. \tag{4}$$

This problem was solved numerically using a Ural-3 computer. The temperatures at the center of the sphere and at the outer boundary of the cell were computed to an accuracy of 0.2%.

The equilibrium temperature is assumed to be

$$\theta_r = \frac{\theta_1 - \theta_2}{\theta_1} \leqslant 0.01. \tag{5}$$

The computed curves for the cooling of a sphere were compared with the experimental results obtained from the apparatus shown in Fig. 2.

The drying cabinet contains a wooden box of dimensions 90 × 90 × 120 mm with metal or gypsum spheres with d_1 between 9.55 and 20.5 mm. The spheres were glued with epoxide resin to porcelain grid gratings with d = 1.1 mm and pitch 25 mm, the mass of which is negligible in comparison with the filling. A copper–constantan thermocouple of 0.1 mm wire was fixed to the central sphere. A constant temperature, 20°C higher than that of the dispersed material, was maintained in the cabinet. The cold thermocouple junction was placed in a small box outside the cabinet. The emf of the thermocouple was measured using an M-16 galvanometer (scale unit 0.1°C). After the spheres were initially heated to the equilibrium temperature (taking 2-3 h) the dispersed materials – fine sand, 101, 274, 632 μ – was poured into the box. Its thermophysical properties are given in Table 1.

The cooling curves for the sphere in one set of conditions were plotted 4-6 times, after which the resulting data were averaged. The scatter of the experimental points did not exceed ±15%.

Calculated and experimental results are compared on Fig. 3. We see from the graphs that the calculated results agree with the experimental results. Also shown in Fig. 3 are the results of calculating the time for the initial heating of the mixture using the method of [1]. We see that calculation using the method of [1] significantly reduces the true time to reach the equilibrium temperature.

Thus, calculation of the temperature change in the mixture using an adiabatic composite sphere and boundary conditions of the fourth kind yields satisfactory agreement with experiment. In this connection, an attempt was made to find an approximate analytic solution of the problem. To do this, the system (4) was reduced to the following form for N = 3, M = 3:

$$\frac{d\theta_1}{d\tau} = b(\theta_2 - \theta_1),$$

$$\frac{d\theta_2}{d\tau} = c(\theta_1 - \theta_2) \quad \text{for} \quad \tau = 0, \quad \theta_1 = 1, \quad \theta_2 = 0.$$

TABLE 1. Thermophysical Characteristics of Sand

Thermophysical characteristics	Sand size, μ		
	101	274	632
a, m²/h	$5{,}9\cdot 10^{-4}$	$7{,}4\cdot 10^{-4}$	$8{,}58\cdot 10^{-4}$
λ, W/m²·deg	0,135	0,206	0,248
c, kJ/deg	0,452	0,452	0,452

Eliminating the right side of these equations and solving by the method of separating the variables, we obtain

$$\theta_1 = \frac{b}{b+c}\{\exp[-\tau(b+c)]-1\}+1,$$
$$\theta_2 = \frac{c}{b+c}\{1-\exp[-\tau(b+c)]\}. \quad (6)$$

Here

$$b = \frac{8a_1\lambda_2}{R_1[\lambda_1(R_2-R_1)+\lambda_2R_1]};\quad c = \frac{8a_2\lambda_1R_1}{(R_2^2-R_1^2)[\lambda_1(R_2-R_1)+\lambda_2R_1]}.$$

If we substitute the condition that θ_1 and θ_2 coincide to within p, we obtain from (6) the time to reach the equilibrium temperature

$$\tau_p = \frac{-\ln p}{b+c}.$$

When $\tau \to \infty$ the equilibrium temperature is defined by the equation

$$\theta_p = \frac{a_2\lambda_1R_1^2}{\lambda_1a_2R_2^2+\lambda_2a_1(R_2^2-R_1^2)}. \quad (7)$$

The equilibrium temperature of the sphere–shell system can also be determined by a different method: from the heat balance equation. Assuming that $\theta_1 = 1$ and $\theta_2 = 0$ for $\tau = 0$, we obtain

$$\frac{4}{3}c_1\rho_1\pi R_1^3 = \frac{4}{3}c_1\rho_1\pi R_1^3\,\theta_r + \frac{4}{3}c_2\rho_2\pi(R_2^3-R_1^3)\theta_r;$$

from which

$$\theta_r = \frac{a_2\lambda_1R_1^2}{\lambda_1a_2R_1^2+\lambda_2a_1\left(\frac{R_2^3}{R_1}-R_1^2\right)}. \quad (8)$$

By comparing (7) and (8) we find that the exact solution differs from the inexact in having R_2^3/R_1 in place of R_2^2 in the denominator of the fraction. Hence, the exact value of the coefficient c has the form

$$c_k = \frac{8a_2\lambda_1R_1}{\left(\frac{R_2^3}{R_1}-R_1^2\right)[\lambda_1(R_2-R_1)+\lambda_2R_1]}.$$

Then

$$b+c_k = \frac{8\left[a_2\lambda_1R_1^2+a_1\lambda_2\left(\frac{R_2^3}{R_1}-R_1^2\right)\right]}{[\lambda_2R_1+\lambda_1(R_2-R_1)]\,R_1\left(\frac{R_2^3}{R_1}-R_1^2\right)}.$$

After similar corrections the computational expressions for θ_1, θ_2, and τ take the following forms:

$$\theta_1 = \frac{b}{b+c_k}\{\exp[-\tau(b+c_k)]-1\}+1, \quad (9)$$

$$\theta_2 = \frac{c_k}{b+c_k}\{1+\exp[-\tau(b+c_k)]\}, \quad (10)$$

$$\tau = -\frac{\ln p}{b+c_k}. \quad (11)$$

Equations (9), (11) are valid for $16 < (d_1/d_2) < 250$; $2 < (a_1/a_2) < 760$. Since the above corrections hold unconditionally only at the point $\tau = \infty$, we can compare the computed values using (9) and (10) with the numerical solution of the same problem (Fig. 4). It follows from the graphs that the approximate analytic solution agrees with the numerical solution of the problem to within ±5%. In this connection, Eqs. (9), (10) can be used to calculate the temperature at typical points of the mixture, while (11) is recommended for calculating the time needed for the components of a uniform mixture of two dispersed materials to reach equilibrium temperature. Then R_2 is determined from the equations

$$R_2 = R_1 \sqrt[3]{\frac{1}{\beta_1}} = R_1 \sqrt[3]{\frac{1}{1-\beta_2}},$$

where

$$\beta_1 = \frac{V_1}{V_1+V_2}; \quad \beta_2 = \frac{V_2}{V_1+V_2}.$$

NOTATION

t_1, t_2, t_1^0, t_2^0	are the current and initial temperatures of the hot and cold media, °C;
T_1^0, T_2^0	are the initial temperatures of the components, °K;
T_k	is the equilibrium temperature, °K;
η_1, η_2	are the volume concentrations of mixture components;
c_1, c_2, ρ_1, ρ_2	are the heat capacities and densities of components;
d_1, d_2	are the dimensions of component particles;
r	is the current value of coordinate;
$a_1, a_2, \lambda_1, \lambda_2$	are the thermal diffusivity and thermal conductivity of components;
V_1, V_2	are the volumes of coarse- and fine-grained materials;
N, M	are the numbers of separation points in the sphere and the spherical shell;
θ_1, θ_2	are the relative nondimensional temperatures of the hot medium at the center of the sphere and of the cold medium at the outer boundary of the elementary cell.

LITERATURE CITED

1. O. A. Tsukhanova and G. D. Salamandra, Izv. Akad. Nauk SSSR, OTN, No. 8 (1947).
2. A. P. Baskakov, Z. Sporysh, and Chu-Hsiang, in: Proc. Inter-University Conference on Industrial Power Utilization and Efficiency of Burning Fine-Grain Fuel [in Russian], Sverdlovsk (1959).
3. N. V. Antonishin, V. V. Lushchikov, and L. E. Simchenko, Inzh.-Fiz. Zh., 13, No. 3 (1967).
4. A. P. Baskakov, Inzh.-Fiz. Zh., 12, No. 5 (1967).
5. A. F. Chudnovskii, Heat Transfer in Dispersed Media [in Russian], GITTL (1954).
6. A. V. Lykov, The Theory of Heat Conduction [in Russian], GITTL (1952).
7. V. V. Kuklinskii, L. P. Knyazev, and Z. P. Gorbis, Papers from the Eighth All-Union Inter-University Conference on Problems in Vaporization, Combustion, and the Gas Dynamics of Dispersed Systems [in Russian], Odessa (1968).
8. O. A. Tsukhanova and G. D. Salamandra, Izv. Akad. Nauk SSSR, OTN, 9, No. 1 (1947).
9. F. Kh. Temlyak, in: Heat and Mass Transfer in Drying and Thermal Processes [in Russian], Minsk (1966).
10. V. K. Solyakov, Scientific Reports of the High Schools [in Russian], No. 3, Énergetika (1958).